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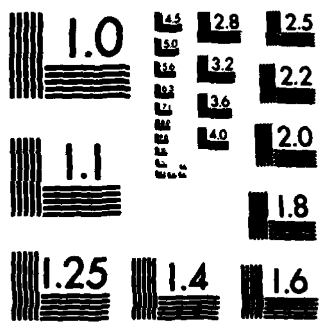

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INPUT-OUTPUT STABILITY ANALYSIS WITH MAGNETIC HYSTERESIS NON-LINEARITY

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Abstract

Popov type frequency domain conditions for stability of feedback systems containing ferromagnetic hysteresis non-linearity are established.

I. Introduction

Popov criterion and its extensions consider non-linear elements that are memoryless and pass through the origin, i.e., $f(0) = 0$. For an important class of non-linearities, ferromagnetic hysteresis, none of the above conditions are satisfied, i.e., it is neither non-dynamic nor pass through the origin. Therefore, to analyze the stability of systems containing this type of non-linearity, appropriate modifications to Popov's approach should be made.

Published material to tackle this problem is scarce. The only work known to us is by Lecoq and Hopkin [1], where by letting the derivative of their input signals to belong to exponentially weighted L_2 spaces, they obtained bounded input-bounded output stability for systems containing hysteresis non-linearities.

In the present paper we analyze the stability of feedback systems of the form shown in fig. (1a), where N is a ferromagnetic hysteresis non-linearity and H is a linear element. The analysis is done by substitution of the model for the hysteresis proposed by Chua and Stromsmoe [2]. Then the concept of passivity is utilized to derive Popov type frequency domain conditions on the linear element H for stability of the feedback system. It will be shown that if the same conditions as in the classical Popov criterion[1] are satisfied by inputs u_1 and u_2 and linear element H then the feedback system of fig. (1a) is stable if the non-linear element N is a ferromagnetic hysteresis.

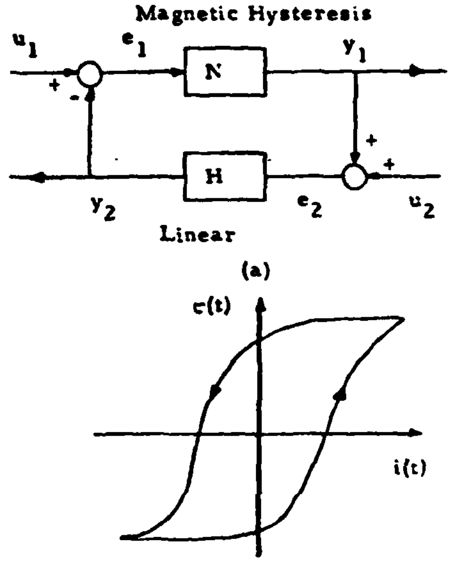
II. Hysteresis Modeling

The model for ferromagnetic hysteresis of Chua and Stromsmoe [2] is given by

$$\frac{dy}{dt} = g\circ[x(t) - f\circ(y(t))] \qquad (2.1)^2$$

where $x(t)$ and $y(t)$ are real-valued, continuous input and output signals of the hysteresis non-linearity

Magnetic Hysteresis



(a)

(b) Ferromagnetic Hysteresis Loop.

Fig. 1

Table 1
Notations

Symbol	Meaning
R, R_+	Field of real and positive real numbers.
L_1	The space of signals such that $\int_{-\infty}^{\infty} \lvert x(t)\rvert dt$ exists.
L_∞	The space of bounded signals.
L_{2e}	The space of signals which are square integrable on every bounded interval $[0, T]$ [9].
L_2	The Hilbert space of signals which are square integrable on $(-\infty, \infty)$ with inner product $< x, y >$.
$< x, y >$	$\int_{-\infty}^{\infty} y^*(t)\, x(t)\, dt$
$\lVert x \rVert$	$\sqrt{< x, x >}$
$< x, y >_T$	$\int_0^T y^*(t)\, x(t)\, dt$
$\lVert x \rVert_T$	$\sqrt{< x, x >_T}$
x_T	Truncated x [6]
$\mathcal{J}$	Set of instances of time of interest.
$\mathcal{A}$	Convolution Algebra [6].
$Re\{\cdot\}$	Real part of a complex quantity.

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[1] By the classical Popov criterion, we refer to the earliest version of the result obtained by Popov [3] and also derived by different approaches in [4], [5], [6], and not later generalizations of the result by Yakubovich [7] among others.

[2] $\circ$: Functional Composition.

representing the current $i(t)$ and the flux linkage $\varphi(t)$ of an inductor (transformer); and g and f are strictly monotonically increasing, differentiable, onto functions enjoying the important property of

$$g(0) = f(0) = 0$$

Equation (2.1) models the behavior of ferromagnetic hysteresis successfully and with very good accuracy. It predicts the expansion of the area of the hysteresis loop with increasing frequency and predicts minor hysteresis loops such as commonly occur when a d-c plus periodic input is applied.

After plotting the hysteresis loop for a convenient signal, simple procedures are given in [2] to determine g and f for that loop. When non-linear functions g and f are determined, they can be substituted in the model of equation (2.1) to predict, with good accuracy, the hysteresis shape and/or its output for any arbitrary input. For further detail and examples see [2].

III. Passivity and Stability

Definition (3.1) [6]: Let $H: L_{2e} \to L_{2e}$. Then H is passive iff there exists some constant $\delta \in R$ such that

$$< Hx, x >_T \geq \delta \qquad \forall x \in L_{2e} \quad \forall T \in \mathcal{J}$$

Definition (3.2) [6]: Let $H: L_{2e} \to L_{2e}$. Then H is strictly passive iff there exists $\delta > 0$ and some constant $\beta \in R$ such that

$$< Hx, x >_T \geq \delta \| x_T \| + \beta \qquad \forall x \in L_{2e} \quad \forall T \in \mathcal{J}$$

Definition (3.3) [11]: The feedback system of fig. (1a) is said to be finite gain L_2-stable if

a) $e_1, e_2, y_1, y_2 \in L_2 \quad \forall\, u_1, u_2 \in L_2$

b) There exists constants ρ_1 and ρ_2 such that
$\| e_1 \|, \| e_2 \|, \| y_1 \|, \| y_2 \| \leq \rho_1 \| u_1 \| + \rho_2 \| u_2 \| \quad \forall u_1, u_2 \in L_2$

In the following well-known theorem, the concept of passivity is used to establish finite gain L_2-stability of feedback system shown in fig. (1a), where N and H are considered to be operators in the general sense.

Theorem (3.1): Consider the feedback system shown in fig. (1a)

$$e_1 = u_1 - He_2$$
$$e_2 = u_2 + Ne_1$$

where $H, N : L_{2e} \to L_{2e}$. Assume that for any $u_1, u_2 \in L_2$ there are solutions $e_1, e_2 \in L_{2e}$. Suppose that there are real constants ν, δ, and ε such that

$$\| Hx \|_T \leq \nu \| x \|_T \tag{3.1}$$

$$< Hx, x >_T \geq \delta \| x \|_T^2 \tag{3.2}$$

$$< x, Nx >_T \geq \varepsilon \| Nx \|_T^2 \tag{3.3}$$

$$\forall x \in L_{2e}, \ \forall T \in \mathcal{J}$$

Under these conditions if

$$\delta + \varepsilon > 0 \tag{3.4}$$

Then the feedback system is finite gain L_2-stable.

Proof: See for example [6].

IV. Main Results

Substitution of the model given by equation (2.1) for the hysteresis non-linearity N gives the feedback system of fig. (2). Note that although the standard magnetic hysteresis non-linearity, as shown in fig. (1b), is the plot of flux linkage ($\varphi(t)$) vs. current ($i(t)$) of an inductor (transformer), but from circuit analysis point of view, the input and output of the model replaced for N as shown in fig. (2) are current through and voltage across the inductor (transformer).[3]

Next, the main stability result is presented.

Theorem (4.1): Consider the feedback sytem of fig. (2), where $h(t) \in L_1(R_+)$[4], and $h(t) \in \mathcal{A}$. Assume that for any $u_1, u_2 \in L_2$, there are solutions $e_1, e_2, y_1, y_2 \in L_{2e}$. If a constant $q \geq 0$ exists such that for some constant δ

$$\mathrm{Re}\, \{(1+qj\omega)\, H(j\omega)\} = \delta > 0 \quad \forall\, \omega \geq 0 \tag{4.1}$$

Then $\forall\, u_1, \dot{u}_1, u_2 \in L_2$

a) (i) $e_1, \dot{e}_1, e_2, y_1, y_2, \dot{y}_2 \in L_2$.

(ii) there exists constants ρ_1 and ρ_2 such that
$\| e_1 \|, \| \dot{e}_1 \|, \| e_2 \|, \| y_1 \|, \| y_2 \|, \| \dot{y}_2 \|$
$\leq \rho_1 (\| u_1 \| + q \| \dot{u}_1 \|) + \rho_2 \| u_2 \|$

i.e., finite gain L_2-stability.

b) $e_1, e_2, y_1, y_2 \in L_\infty$ are continuous, and go to zero as $t \to \infty$.

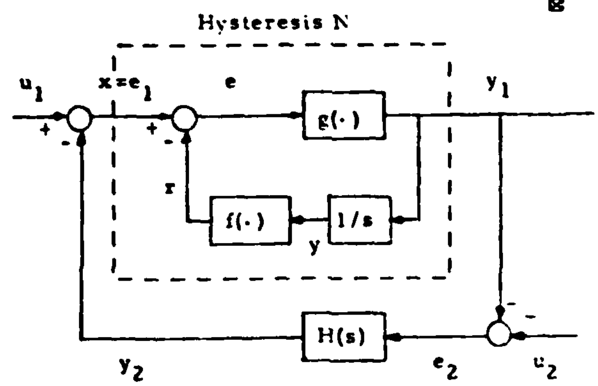

Fig. 2

Proof: See appendix.

Upper and lower bounds of non-linearities g and f can be taken into consideration to obtain less conservative classes of linear element $H(s)$. As an example, an upper bound on g is exploited in the

[3] The voltage across an inductor (transformer) is proportional to rate of change of the flux linkage, constant of proportionality being the number of turns.

[4] $h(t)$ is the impulse response of $H(s)$.

following Corollary.

Corollary (4.1): Consider the feedback system of fig. 2, where $g \in$ sector $(0,k)$, $h(t) \in L_1(R_+)$, $\dot{h}(t) \in \mathcal{A}$. Assume that for any $u_1, u_2 \in L_2$, there exists solutions $e_1, e_2, y_1, y_2 \in L_{2e}$. If a constant $q \geq 0$ exists such that for some constant δ

$$\mathrm{Re}\{(1+qj\omega)H(j\omega)\} + \frac{1}{k} = \varepsilon > 0 \quad \forall\, \omega \geq 0 \qquad (4.2)$$

then $\forall\, u_1, \dot{u}_1, u_2 \in L_2$ conclusions of Theorem (4.1) hold. ∎

Proof: For outline of the proof, see appendix. ∎

V. Conclusion

Popov type frequency domain conditions for stability of feedback systems containing ferromagnetic hysteresis non-linearity are established. To obtain the results, model of Chua and Stromsmoe [2] for hysteresis is employed and the concept of passivity is utilized.

VI. Appendix

To simplify the proof of Theorem (4.1) and Corollary (4.1), the following two lemmas will be proved first.

Lemma (A.1): Let $q \geq 0$, $g \in$ sector $(0,\infty)$. Then the system of fig. (A.1) is passive [6]. ∎

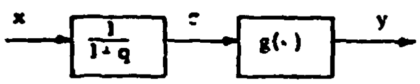

Fig. (A.1)

Proof: $\langle y, x \rangle_T = \langle g(\sigma), \sigma + q\dot{\sigma} \rangle_T$

$$= \langle g(\sigma), \sigma \rangle_T + q \langle g(\sigma), \dot{\sigma} \rangle_T$$

(i): $\langle g(\sigma), \sigma \rangle_T \geq 0$ because $g \in$ sector $(0,\infty)$

(ii): $q \langle g(\sigma), \dot{\sigma} \rangle_T = q \int_0^T g(\sigma)\, \dot{\sigma}\, dt$

$$= q \int_{\sigma(o)}^{\sigma(T)} g(\sigma)\, d\sigma$$

Define $G(\sigma) = \int_o^{\sigma} g(\xi)\, d\xi$, where $G: R \to R$.

Clearly $G(\sigma) \geq 0 \quad \forall\, x \in R$. Then

$q \langle g(\sigma), \dot{\sigma} \rangle_T \geq -q\, G[\sigma(o)] \quad \forall\, T \geq 0$

(i) and (ii) implies $\langle y, x \rangle_T \geq -q\, G[\sigma(o)] \quad \forall\, T \geq 0 \;\; \forall\, x \in L_{2e}$.

Passivity follows. ∎

Lemma (A.2): Let $q \geq 0$, $f \in$ sector $(0,\infty)$, $\frac{df(x)}{dx} > 0$. Then the system of fig. (A.2) is passive. ∎

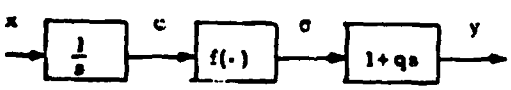

Fig. (A.2)

Proof: $\langle y, x \rangle_T = \langle \sigma + q\dot{\sigma}, \dot{c} \rangle_T$

$$= \langle f(c), \dot{c} \rangle_T + q \left\langle \frac{d}{dt}[f(c)], \dot{c} \right\rangle_T$$

(i) $\langle f(c), \dot{c} \rangle_T \geq -F[c(0)] \quad \forall\, T \geq 0$

where $F: R \to R$, by lemma (A.1,(ii)).

(ii) $q \left\langle \frac{d}{dt}[f(c)], \dot{c} \right\rangle_T = q \int_0^T \frac{d}{dt}[f(c)] \frac{dc}{dt}\, dt$

$$= q \int_0^T \frac{d}{dc}[f(c)] \left[\frac{dc}{dt}\right]^2 dt \geq 0$$

because $\frac{d}{dc}[f(c)] > 0$ and $\left[\frac{dc}{dt}\right]^2 \geq 0$

(i) and (ii) implies that $\langle y, x \rangle_T \geq -F[c(0)] \;\; \forall\, T \geq 0 \;\; \forall\, x \in L_{2e}$

Passivity follows. ∎

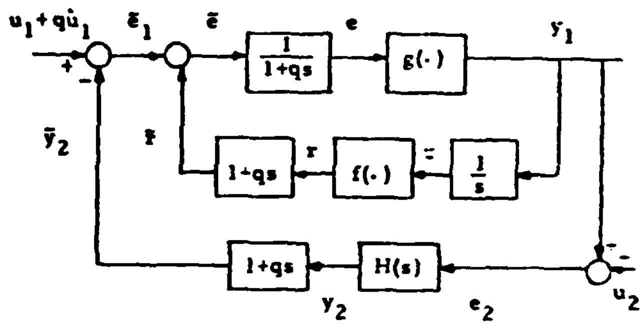

Fig. (A.3)

Proof of Theorem (4.1): (a): By inclusion of the multiplier $(1+qs)$, $q \geq 0$, transform the feedback system of fig. (2) to the one shown in fig. (A.3).

(i): $\langle y_1, \bar{e}_1 \rangle_T = \langle y_1, \tilde{e} \rangle_T + \langle y_1, \tilde{f} \rangle_T$

(ia): $\langle y_1, \tilde{e} \rangle_T \geq -q\, G[e(0)]$ by lemma (A.1)

(ib): $\langle y_1, \tilde{f} \rangle_T \geq -q\, F[c(0)]$ by lemma (A.2)

Then (ia) and (ib) implies that feed forward block is passive.

(ii) $(1+qs)H(s)$ is strictly passive if inequality (4.1) is satisfied. Since $h(t) \in L_1(R_+)$ and $\dot{h}(t) \in \mathcal{A}$, $(1+qs)H(s)$ has finite gain.

Therefore, by Theorem (3.1) the feedback system of fig. (A.3) is finite gain L_2-stable $\forall\, u_1, \dot{u}_1, u_2 \in L_2$, i.e., $\bar{e}_1, e_2, y_1, \tilde{y}_2 \in L_2$ and $\|\bar{e}_1\|, \|e_2\|, \|y_1\|, \|\tilde{y}_2\| \leq c_1 \|u_1 + q\dot{u}_1\| + c_2 \|u_2\|$.

From Fig. (A.3), $y_2(t) = m(t) * \tilde{y}_2(t)$ where $m(t) = L^{-1}\left\{\frac{1}{1+qs}\right\}$[5]. $m(t) \in L_1$ and $\tilde{y}_2(t) \in L_2$, therefore, $y_2(t), \dot{y}_2(t) \in L_2$ [6, Appendix C]. Furthermore, $\|y_2(t)\|_{L_2} = \|m(t) * \tilde{y}_2(t)\|_{L_2} \leq \|m(t)\|_{L_1} \|\tilde{y}_2(t)\|_{L_2}$ [6, App. C].

But $\|m(t)\|_{L_1}$ is finite (= Constant C).

Therefore

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[5] $L^{-1}\{\cdot\}$: Inverse Laplace Transform.

$$\|y_2(t)\|_{L_2} \le C\|\bar{y}_2(t)\|_{L_2}$$
$$\le C\rho_1'\|u_1 + q\dot{u}_1\|_{L_2} + C\rho_2'\|u_2\|_{L_2}$$
$$\le C\rho_1'(\|u_1\|_{L_2} + q\|\dot{u}_1\|_{L_2}) + C\rho_2'\|u_2\|_{L_2}$$

On the other hand, $\dot{y}_2(t) = \dot{m}(t) * \bar{y}_2(t)$. Then

$$\|\dot{y}_2(t)\|_{L_2} \le \|\dot{m}(t)\|_{L_1}\|\bar{y}_2(t)\|_{L_2}$$
$$\le C'\rho_1'(\|u_1\|_{L_2} + q\|\dot{u}_1\|_{L_2}) + C'\rho_2'\|u_2\|_{L_2}$$

because $\|\dot{m}(t)\|_{L_1}$ is finite too.

Similarly, $e_1(t) = m(t) * \bar{e}_1(t)$. Therefore, similar conclusions for $e_1(t)$ follow immediately.

(b): $y_2, \dot{y}_2 \in L_2$ and $e_1, \dot{e}_2 \in L_2$ implies that $y, e \in L_\infty$, are continuous, and go to zero as $t \to \infty$ [11]. Since the model, i.e., equation (2.1), is a continuous mapping from input to output [2], therefore, $e_1 \in L_\infty$, and $e_1(t) \to 0$ as $t \to \infty$ implies that the same properties hold for $y_1(t)$, i.e., $y_1(t) \in L_\infty$, is continuous, and go to zero as $t \to \infty$.

Similar conclusions for e_2 are immediate. ∎

Proof Outline of Corollary (4.1): Apply a positive feedback of gain $\frac{1}{k}$ around g. To compensate for it, apply a positive feed forward with gain 1/k to H(s). Let $\hat{g} = (g^{-1} - \frac{1}{k})^{-1}$. Then $\hat{g} \in$ sector $(0, \infty)$ and $\hat{g}(0) = 0$. Following the same procedure as Theorem (4.1). Conclusions are immediate. ∎

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